Mobile Learning Game Improves 5th Graders' Fractions Knowledge and Attitudes

Prepared by
Prof. Michelle Riconscente
University of Southern California
riconsce@usc.edu

Published by
GameDesk Institute
2332 Cotner Ave, Suite 203, Los Angeles, CA 90064
www.gamedesk.org | ContactUs@gamedesk.org

This study was funded by Motion Math and made possible by a grant from the Noyce Foundation.

TABLE OF CONTENTS

Executive Summary 1

Acknowledgements 2

Introduction 3

Study Design and Hypotheses 9

Methods 11

Participants 11

Study Measures 12

Procedures 15

Results 16

Fractions Knowledge 16

Fractions Attitudes: Self-Efficacy, Liking, and Knowledge Self-Ratings 19

Participants' Ratings of Motion Math 21

Discussion 23

Limitations 27

Recommendations 28

Conclusion 31

References 32

APPENDIX A: School A Fractions Knowledge Test 35

APPENDIX B: School B Fractions Knowledge Test 41

EXECUTIVE SUMMARY

A controlled experimental study was conducted with 122 children to determine whether Motion Math, a fractions game designed for the iPad, iPhone, and iPod, improves their fractions knowledge and attitudes. Although fractions knowledge is essential for future success in mathematics, national data show that the vast majority of US students fail to become proficient in fractions. With the advent of mobile technologies such as iPad tablets, new kinds of interactions with subject matter have become possible that have potential for improving learning. However, to date no experimental studies have been conducted to determine the efficacy of any iPad app for improving children's knowledge. As the first controlled study of an educational iPad app, the research presented in this report marks an important step forward for documenting the potential of new technologies to support learning and engage learners in challenging academic subjects.

Key Findings

- Children's fractions test scores improved an average of over 15% after playing Motion Math for 20 minutes daily over a five-day period, representing a significant increase compared to a control group.
- Children's self-efficacy for fractions, as well as their liking of fractions, each improved an average of 10%, representing a statistically significant increase compared to a control group.
- All participants rated Motion Math as fun and reported wanting to play it again; nearly all (95%) children in the study reported that their friends would like the game, and that the game helped them learn fractions.
- Taken together, the data from this experimental study offer solid evidence that Motion Math successfully integrates entertainment value with educational value.

ACKNOWLEDGEMENTS

Several individuals made important contributions to this study. Professor Steven Hecht, now at Nova Southeastern University, provided excellent suggestions regarding the study measures and design. Many thanks to Lucien Vattel for sharing invaluable research and game design insights, and to GameDesk for assistance with site implementation. I am grateful to Paul Bailey of Western Center Academy for supporting this project above and beyond the call of duty. Warren Buckleitner of Children's Technology Review and Gregory Ferenstein of Fast Company provided helpful feedback on an earlier draft of this report. Special thanks to Jose Franco and Mayra Lavadenz, and to all the children, teachers and administrators whose dedication and gracious participation made this project possible.

The teaching of fractions has been in crisis for over fifty years.
Wu (2010)

INTRODUCTION

The topic of fractions has long been a challenging one for children (Hiebert, 1985; Newton, 2008; Wu, 2010). Research by the National Center for Educational Statistics (NCES) shows that only 13% of United States fifth graders are proficient in fractions (Princiotta, Flanagan, & Germino Hausken, 2006). Even though the topic of fractions appears early in children's mathematics trajectory, success or failure at this juncture carries long-term consequences. The extent to which students master fractions is a strong predictor of later mathematical success, particularly in algebra, which serves a "gatekeeper" role for access to higher education (US Department of Education, 1997).

It is in this context that instructional designers are increasingly turning to new technologies and creating learning experiences that leverage the unique interactive features of devices such as the iPad. Although there are hundreds of iPad apps on the market that claim to improve learning, to date no controlled study has tested the effectiveness of any educational iPad app. This report describes a controlled study of the iPad app Motion Math, a game aimed at improving students' ability to relate various fraction representations to the number line. The developers of the game believe that by gaining skill at this central aspect of number sense, students will develop a better understanding of fractions that will transfer to improved performance on the kinds of questions posed on state and national standardized tests. Although little research has examined the relation between learning and iPad interactions, from a learning science perspective, there are several reasons to expect that the experience of playing Motion Math could foster learning. In the game, which relies on the "tilt" feature of the iPad, the player is continuously presented with fractions problems to solve, instant feedback, rewards, and increasing levels of challenge. The question is whether this combination of experiences does more than help players get better at the game itself. The present study sought to determine whether the game increased children's fractions knowledge, improved their attitudes toward fractions, and was perceived as fun and helpful by participants.

Review of the Literature

In the United States, most students are initially introduced to fractions in the second or third grade. By fifth grade they are learning how to perform arithmetic operations such as addition and subtraction with fractions. However, as evidenced by the NCES figure cited earlier, the instructional

approaches engaged in most classrooms are not proving effective for the vast majority of students. Research shows that classroom instruction tends to focus on fractions concretely as "parts of a whole" (Misquitta, 2011). This instruction tends to generate an inaccurate conceptualization which subsequently becomes an obstacle to students' abstract reasoning about fractions. For instance, it is difficult for students to reconcile the part-whole model with the fact that fractions are continuous and infinitely divisible (Behr, Lesh, Post, & Silver, 1983; Hiebert & Tonnessen, 1978). Moreover, students' prior knowledge of whole numbers has been theorized to lead to misinterpretations of the representational form of numerator over denominator, which students often understand simply as two whole numbers (Gallistel & Gellman, 1992).

> The model for introduction currently in most of our textbooks is that of regions (square, circle, line). This enables us to talk of shares, but the result is a tangible amount (slice of pizza, cube of chocolate) which does not neatly fit within the operations of addition, subtraction, multiplication and division. How can you multiply two pieces of pizza? (Hart, 2000, 53-54)

Understanding fractions abstractly means recognizing that fractions are numbers, and as such they can be placed on a number line. Yet most children fail to develop this knowledge (NMAP, 2008; Siegler, Thompson, & Schneider, 2011), as can be seen in their reliance on procedural knowledge rather than on conceptual understanding when solving problems with fractions (Wong & Evans, 2007).

Case and colleagues have suggested that the mental number line is essential to early numerical understanding (Case & Griffin, 1990; Case & Okamoto, 1996). Siegler and Ramani (2009) theorized that the mental number line facilitates arithmetic problem-solving by providing a "retrieval structure that improves encoding, storage, and retrieval of numerical information by organizing the information around the numbers' magnitudes" (p. 555). Likewise, renowned mathematician Wu has published widely on the value of teaching children the relation between fractions and the number line.

> [T]he use of the number line has the immediate advantage of conferring coherence on the study of numbers in school mathematics: decimals are rightfully restored as fractions of a special kind, and positive and negative fractions all become points on the number line. In particular, whole numbers are now points on the number line too and ***the arithmetic of whole numbers, in this new setting, is now seen to be entirely analogous to the arithmetic of fractions***. (…) It must be said that this coherence has been largely absent from school mathematics for a long time. (Wu, 2008, p. 4; emphasis original)

In a recent report to the US Department of Education's Institute of Education Sciences, Siegler and colleagues (2010) reviewed the available research on fractions teaching and learning. Among the five recommendations they forwarded for effective fractions instruction is the suggestion to "[h]elp students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward" (p. 19). The same report emphasizes that there is a dearth of rigorous experimental studies that examine the effectiveness of specific approaches to fractions instruction. With postsecondary success so closely bound to early mathematics achievement, identifying research-based ways to support students' fractions learning is of highest priority (US Department of Education, 1997; Wu, 2008).

Importantly, children's struggles with understanding fractions often go hand-in-hand with low motivation and negative attitudes toward mathematics (Ashcraft, 2002; Stipek, Salmon, Givvin, Kazemi, Saxe, & MacGyvers, 1998). Numerous studies of students' motivation have shown that low self-efficacy—i.e., lack of confidence in one's ability to successfully complete academic tasks—is associated with lower achievement (Bandura, 1997; Pajares, 1996). Research also documents the importance of expectations for success and valuing of academic tasks for increased persistence (Wigfield & Eccles, 2000). Moreover, children are more likely to develop long-term interest in material that they understand and see as relevant (Hidi & Renninger, 2006; Riconscente, 2010). It is thus reasonable to conclude that comprehensively addressing the fractions crisis will require approaches that improve students' fractions knowledge while simultaneously transforming negative attitudes into positive ones.

Successfully integrating learning with motivation is a long-standing hallmark of computer-game design. In computer games, players are constantly challenged by increasingly difficult obstacles they must overcome to attain a goal. The kinds of obstacles they face are new, even counterintuitive, and players must discover the rules of play in order to win. Good games will "ramp" up the challenge, by adding new rules and constraints that must be figured out and surpassed. Failure is accepted as a natural part of game play. In contrast to most academic settings, where failure is rarely met with increased enthusiasm, games are designed such that failure and challenge strengthen motivation.

The last decade has seen an exponential surge in the creation of educational games to address learning challenges in a variety of domains (Habgood & Ainsworth, 2011). The theory and study of educational games is increasingly taking center stage at academic conferences, and several

journals dedicated to games and learning have sprung up. Features of new technologies, especially mobile devices such as the iPad and iPhone, offer instructional designers more options for creating effective learning experiences that move beyond static presentation, limited interaction, and the walls and schedules of formal schooling. For example, the multipoint touch-sensitive iPad display can be 'pinched' to zoom in or out on an image. The iPad also detects when it is being moved or tilted, and through GPS data knows its position on the Earth.

Some researchers have leveraged the capability of these new technologies to create embodied learning experiences. This approach is grounded in the insight that "cognitive processes are deeply rooted in the body's interactions with the world" (Wilson, 2002). In other words, rather than residing entirely in one central location—the brain—many cognitive scientists are now exploring the ways that knowledge is rooted in our physical interactions with the world (Clark, 1999; Wilson, 2002). Recent studies have examined embodied cognition in relation to mathematics. For example, Alibali and Nathan (2011) found that teachers' and students' gestures were valid evidence of their mathematical understanding.

Scholars have begun to apply the theory of embodied cognition to create *embodied learning experiences*. For example, Ramani and Siegler (2008) showed that preschoolers gained mathematical knowledge by playing board games. They theorized that opportunities to physically interact with a number line by moving tokens along linear board games helped children to develop a mental number line by offering tangible clues about the order and magnitude of numbers. Large-scale virtual embodied installations have also been shown to significantly increase students' understanding of such topics as chemical titration (Tolentino, Birchfield, Megowan-Romanowicz, Johnson-Glenberg, Kelliher, & Martinez, 2009) and geoscience (Johnson-Glenberg., Birchfield., Savvides, Megowan-Romanowicz, 2010).

As promising as these new technologies are, educational games remain challenged by a tension between entertainment and learning. Historically, one or the other has taken precedence in the design process, resulting in games that are either entertaining but not educational, or educational but not fun to play (Vattel, in preparation). Habgood and Ainsworth (2011) suggest that successful educational games are those that establish an intrinsic link between a core game mechanic and the target learning content. In other words, the game play itself engages the player with the content (Malone & Lepper, 1977). This design perspective stands in contrast to many edutainment titles, in which the educational content is artificially injected into game play—as, for instance, when the player must solve a math problem to pass through doors in a castle. Unfortunately this extrinsic

approach to game design is still prevalent. A recent search yielded 115 apps and games developed for the iPad that purport to teach fractions. Upon closer examination, however, many of these "games" are better characterized as math quizzes; others contain explicit instruction, however nearly always the instruction and the game (quiz) are isolated from one another. Importantly, although one unpublished study was found on an iPod literacy game for young children, to date there has been no experimental research to ascertain whether these apps promote children's learning of mathematics.

To extend the literature reviewed here, the present study used a controlled experimental design to test whether the mobile learning game Motion Math improves children's fractions knowledge and attitudes. The most common use of the game is by individuals on their own without the assistance of parents or teachers. Therefore, rather than compare Motion Math to other apps or to classroom instruction, this study focused on whether or not Motion Math achieves its goals as a learning app. In other words, when young people play Motion Math, do they get better at fractions and come away with more positive attitudes toward fractions?

About Motion Math

Created by graduates of Stanford University's Learning, Design, and Technology program, Motion Math was designed to help children strengthen their understanding of the relationship between fractions, proportions, and percentages to the number line. The game plays on iPad, iPhone and iPod devices, leveraging the "accelerometer" feature. In Motion Math, the player tilts the device to direct a falling star to the correct place on the number line at the bottom of the screen (See Figure 1). The stars fall one at a time, and each displays either a fraction, percent, decimal, or pie shape. The correct response generates a rewarding audio and visual response; wrong answers trigger increasingly strong instructional hints, starting with an arrow pointing either left or right, moving to hatch marks that break the number line into the appropriate number of parts, and finally labels on the hatch marks (See Figure 2).

Figure 1. Star falling from top of screen. Player must tilt the device so the star falls at the correct location on the number line.

The game has three levels of difficulty, from beginner to hard, and each game difficulty

Figure 2. Wrong answers trigger increasingly explicit hints. This screen shows the last hint the player receives before losing the level.

Figure 3. Example of a "Less-Equals-More" level. Player must tilt the device so that the falling satellite lands in the correct zone on the number line.

mode consists of 24 increasingly challenging levels. Levels are generated according to an algorithm that increases the range of the denominator and changes the range of the number line. The visual and symbolic representations of the fractions also vary, and during more challenging game play representations are mixed together within each level. After completing every five levels or so, the player is presented with a "Less – Equals – More" challenge (See Figure 3). A comparison value appears on an unmarked number line above a bin marked "Equals". To the left and right of the Equals bin are a "Less" bin and "More" bin, respectively. For each fraction that falls from the sky, the player must tilt the device to drop it in its proper location. Audio, haptic, and visual feedback let the player know whether or not she has hit the mark.

From an instructional design perspective, Motion Math has several promising features. First, unlike most math "games" in which the content is interjected artificially, or learning is isolated from assessment and feedback, Motion Math makes fractions the focus of the game, and integrates feedback directly within game play for a continuous interactive experience. The visual interface, tone, and game mechanics were also designed to promote players' positive attitudes toward fractions.

It is important to note that the Motion Math game design assumes a baseline of prior fractions knowledge. Specifically, the player must already have a basic understanding of the fraction representational form. In other words, Motion Math was not created with the goal of introducing the player to fractions. Rather, it focuses on developing and strengthening a player's

understanding of how fractions are related to the number line. The expectation is that by fostering children's understanding of the relation between fractions and the number line, the game will lead to a better understanding of fractions, which in turn will transfer to fractions knowledge more generally.

STUDY DESIGN AND HYPOTHESES

Many important research questions can be asked regarding new learning technologies. These questions include whether the technology application promotes learning when used on its own, whether it promotes long-term engagement in subject matter, whether it is more effective than traditional forms of instruction, and how it compares to similar learning technologies. Other questions focus on which features of the technology innovation are the keys to its success, which conditions of use lead to the strongest outcomes, and whether certain populations benefit more than others from the application. For producers and consumers alike, answers to each of these questions are valuable. However, they cannot all be asnwered with one study, since customized research designs are necessary for obtaining valid answers to each question.

This purpose of this study was to ascertain whether Motion Math increases learners' fractions knowledge and attitudes, and was intentionally chosen over other purposes for several reasons. First, Motion Math was not designed to replace classroom instruction by a teacher, and therefore it would not have been appropriate to compare the game to classroom instruction. Second, comparing Motion Math to another fractions app would only enable conclusions relative to that one specific app, not to fractions apps more generally. At the time of this study, no apps were available that were sufficiently aligned with the same goals as Motion Math to represent an appropriate and valuable comparison point. Third, since this was the first study to test the effectiveness of any iPad app for learning, the most fundamental question was whether the app indeed attains its goal of helping players understand fractions better.

Data gathered from Motion Math game play and from play tests showed that most individuals improve at the game the more they play. The salient question was therefore not whether players improved at the game itself, but rather whether exposure to the game translates to an increase in fractions knowledge, and whether children can transfer that increased fractions knowledge to questions typically administered on standardized tests Moreover, since the makers of Motion Math also sought to create a game that helped learners develop more positive attitudes

toward math, it was important to determine whether the game successfully promotes learning while also making math appealing.

Table 1. *Study Design: Repeated Measures Crossover Design*

Group	Day 1	Days 1-5	Day 6	Days 6-10	Day 10
1	**Pretest**	*Motion Math*	**Midtest**	control	**Posttest**
2		control		*Motion Math*	

The strongest research designs are those in which the only characteristic distinguishing the two groups is their exposure to the intervention being tested. The present study implemented this kind of design by applying a repeated-measures crossover approach. A repeated-measures crossover design ensures that any differences in test score changes between the two groups can be attributed to the intervention, which in this study was playing Motion Math daily for 20 minutes over five consecutive school days. By having both groups take turns as the experimental (Motion Math) and control (no Motion Math) groups, it is possible to isolate the effects of Motion Math from the potential influence of other characteristics such as prior levels of fractions knowledge, self-efficacy, and fractions liking.

As shown in Table 1, this design uses two groups of children. For the first half of the study, one group serves as the treatment group and the other as the control group. Halfway through the study, the two groups switch. If the hypotheses are supported, the first and second groups should start off with comparable levels of knowledge and attitudes. At the halfway point, the first (Motion Math) group should have significantly higher scores than the second (control) group. After the groups cross over, and the original control group has been exposed to the treatment, their scores should rise to meet those of the original treatment group.

Three overarching research questions were investigated in this study.

- Does playing Motion Math lead to increases in children's fractions knowledge?
- Does playing Motion Math lead to more positive attitudes toward fractions?
- Do children develop positive attitudes toward the Motion Math game itself?

The first and second questions were tested with a set of hypotheses made possible by the study design. The first set of hypotheses pertained to within-group changes. For each group, it was

hypothesized that children's scores would increase significantly after playing Motion Math, but remain constant over the control period. In addition, children's posttest scores within each group were hypothesized to be significantly higher that their pretest scores. Hypotheses were also advanced regarding anticipated differences between groups. Both groups were hypothesized to have equivalent pretest and posttest scores, however midtest scores for Group 1 were expected to be significantly higher than those of Group 2. Since change in scores between groups would provide a more robust test of this research question, it was also hypothesized that Group 1 and Group 2 midtest scores would differ after controlling for pretest scores. The final hypothesis was that the score trajectories from pretest to midtest to posttest would differ significantly between groups. The third research question was examined descriptively, and anticipated that participants would respond positively to the game.

It bears mentioning that the reason the study was conducted in a school setting was not to compare Motion Math to classroom instruction. Rather, conducting the study in a classroom made it possible to control the frequency and amount of time students played the game, to ensure comparable conditions of game play, and to ensure that adults did not intervene while children played the game. Because the purpose of the study was to test whether Motion Math on its own leads to gains in learning and fractions attitudes, it was important to minimize the effect that classroom instruction might have on students' fractions knowledge. Therefore, participating teachers agreed to refrain from teaching fractions to participating students for the duration of the study.

METHODS

Participants

Participants were 122 fifth graders enrolled in two schools in southern California. Whereas a small study would typically include around 30 participants, this study involved many more students based on a power analysis as described by Cohen (1992). To facilitate interpretations regarding the generalizability of results, details are provided here about the two schools. School A serves two equally sized classes of fifth graders, and School B has five fifth grade classes. At each school, students are randomly assigned to classes. Therefore any differences among students should be equally distributed across all classes. School A is an urban public school serving primarily Latino students from low-income homes; School B is a public elementary school serving mostly Caucasian and Latino students. All students at School A have the same math teacher whereas the two classes of students who participated from School B are taught by different math teachers. Students had all

received fractions instruction in fourth grade. During the study, which took place at the start of the 2011-12 school year, the teachers refrained from covering fractions in their mathematics classes. A summary of the two school sites is presented in Table 2. The majority of participants (84%) completed all three measures. Twenty children were absent on at least one day of testing, and were dropped from the analysis due to incomplete data. Seven of these participants were from School A and 13 were from School B.

Table 2. *Summary of Study Sites*

SITE	Group		Ethnicity†			% Low SES*†	CST % proficient or above†		Math Teacher for Group 1 & 2	Test Mode
	1	2	African American	Caucasian	Latino		Math	ELA		
A	29	30	—	—	100	80	40	65	Same	Paper; Timed
B	32	31	5	45	45	60	65	65	Different	iPad; Untimed

† To protect the identity of the study sites, all demographic data were rounded to the nearest 5%.

*Low SES defined as eligible for the free or reduced-price lunch program

Study Measures

Fractions Knowledge. To assess fractions knowledge, a fractions test was created based on released items targeting fractions and number sense from the California Standards Test (CST), the National Assessment of Educational Progress (NAEP), and Trends in International Mathematics and Science Study (TIMSS). Items were created to test children's abilities to solve problems that ranged from very similar to very different from the kinds of problems posed in Motion Math.

At School A, participants completed a paper version of the test, which comprised 34 items. Figure 4 shows a sample item adapted from a NAEP released item. This item was used in both the paper and iPad versions of the test. Test items are presented in Appendix A. Scoring of the paper tests was done by hand using visual estimation. The criteria for correct responses for the paper tests was based on comparison to key positions on the number line. For instance, to earn credit for the question asking the participant to place 0.60 on the number line, it had to be visually slightly to the right of the halfway point and visually left of three-quarters. Some paper questions asked children to place three different fractions on the same number line using symbols (e.g., On the number line

below, draw a square to show where 1/2 is, draw a triangle to show where 1/3 is, and draw a circle to show where 1/4 goes). These items were scored based on correct ordering of the symbols as well as on location of placement on the number line. All items were scored dichotomously.

ORIGINAL NAEP ITEM

On the portion of the number line below, a dot shows where 1/2 is. Use another dot to show where 3/4 is.

ADAPTED ITEM

On the number line below, a shows where $\frac{1}{2}$ is.

Draw another to show where $\frac{3}{4}$ is.

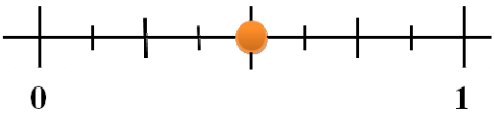


Figure 4. Example of original NAEP item and the adapted item administered in the present study.

At School B, participants completed the tests using the iPad. The iPad version of the test utilized a subset of 26 of the items due to the inability of the computerized interface to render certain question types. Scoring of iPad tests was conducted electronically; students were required to respond to each question before they could proceed to the next screen. Criteria for the online "place a dot" items was scored with a specific algorithm rather than visually. The number line was divided into 46 segments, each seven pixels wide. At seven pixels, children could make precise selections that were still visually distinct. Participants tapped on a dot to indicate their response, and could revise their answer before moving forward. Children received full credit for placement within 3 dots of the exact response. On questions for which the correct answer was the endpoint of the scale (i.e., no visual estimation was necessary) participants only received credit for a precise response.

Fractions Attitudes. The fractions attitude measure comprised three subscales: fractions self-efficacy ("I am good at fractions"; "I can solve hard fractions problems"; "I know a lot about fractions"; "I am good at learning fractions") fractions liking (e.g., "Fractions are fun"; "I like fractions") and fractions-number line knowledge (e.g., "I know where 1/2 goes on the number line"; "I know where 0.65 is on the number line"; "I know where 5/8 is on the number line"). The order of negative and positive response options was mixed to reduce response set threat to validity.

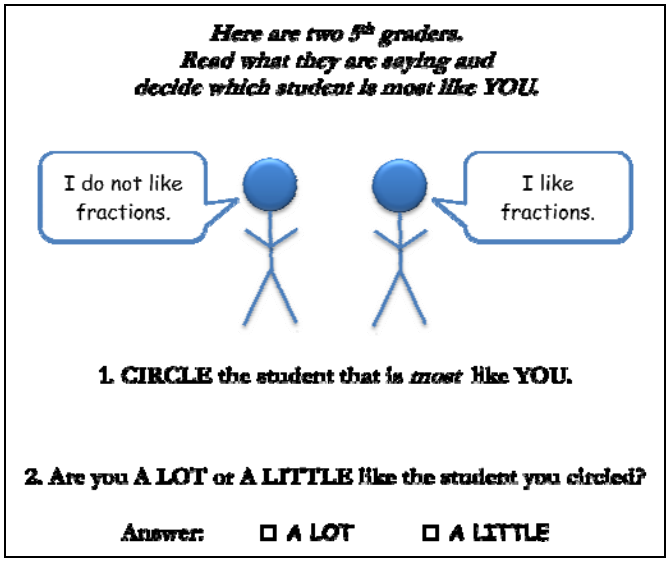


Figure 5. Example question from the Fractions Liking scale.

The attitudinal questions were presented in a format adapted from Susan Harter's (1981) work with young children to avoid social desirability threats to validity. As shown in Figure 5, for each question participants were presented with two stick figures expressing opposite opinions about fractions and asked to circle the student who was most like them. Children then indicated whether they were "a lot" or "a little" like the student they had selected. Responses were coded into a four-point Likert style scale corresponding to strongly disagree (1) through strongly agree (4), and reverse-coded if appropriate. Reliabilities are presented in Table 3.

Game Ratings. Four additional questions, using the same format as the fractions attitudes measures, were used to assess participants' perceptions of the Motion Math game itself. Children indicated whether they wanted to play the game again, whether the game was fun or boring, whether they thought their friends would like the game, and whether the game helped them learn fractions.

Table 3. *Fractions Attitudes Scale Reliabilities*

	Cronbach's Alpha		
Scale	**Pretest**	**Midtest**	**Posttest**
Fractions Self-Efficacy	.77	.84	.81
Fractions Liking	.64	.80	.71
Fractions Knowledge Self-Ratings	.58	.85	.81

Procedures

This study used a repeated measures crossover design. At each study site, one class was randomly assigned to either Motion Math ("Group 1") or control ("Group 2") for the first week. In week two, the order switched. As shown in Table 3, all participants took the test three times, at the start, midpoint, and end of the study period. In this report, these three time points are referred to as pretest, midtest, and posttest, respectively. At both study sites, participating teachers agreed to refrain from teaching topics directly related to fractions during the study, since the purpose of the study was not to compare game play to classroom instruction but rather to ascertain whether the game is effective as a stand-alone instructional tool.

At School A, all participants completed paper-and-pencil tests on the first day of the study. They were instructed via a "STOP HERE" page in the booklet, as well as by their teacher, to wait after completing the attitudinal questions. When all participants were ready, the teacher began the timed section of the test. Children then had 10 minutes to complete the fractions items. School B followed a similar procedure, except that participants completed the measures using the iPads and were not timed. At both sites, for the next five days, students in Group 1 spent 20 minutes daily playing Motion Math. Children in Group 2 served as the control. After five school days, all participants completed all measures under the same conditions as on day one. The groups then switched treatment conditions. For the next five school days, children in Group 2 played Motion Math for 20 minutes daily while Group 1 participants served as the control. After Group 2 had played the game daily for five school days, all participants completed the measures for a third and final time. As a result, at the end of the study all participants had experienced the same amount of exposure to Motion Math.

RESULTS

This study used repeated measures design with staggered treatment, also known as a crossover design. Statistically, this research design corresponds to a 2 x 3 split-plot factorial design, with two treatment conditions (Motion Math, control) and three time points (pretest, midtest, posttest). To respond to the research questions, analyses were conducted to compare the groups to each other over time (i.e., between-subject comparisons), and to compare changes in scores within each group over time (i.e., within-subject comparisons). Data for the fractions attitude and game rating portions of the survey were aggregated across sites since all items were identical. However, to account for different administration conditions for the fractions knowledge tests, those data are presented separately for School A and School B.

Fractions Knowledge

School A Results

At School A, participants completed the test on paper, and were given 10 minutes to complete the fractions test portion of the packet. The teacher observed that on the midtest and posttest, the children in both groups competed with one another to see how many questions they could answer. This resulted in higher scores overall regardless of assignment to treatment or control condition. Since the change in scores from pretest and midtest could therefore not be validly attributed to treatment condition, the data were reviewed for a reasonable cut point. At pretest, there was a clear cut point in both groups after item 20. Ten participants in Group 1, and 16 participants in Group 2, attempted and correctly responded to item 20. However, only two children from Group 1 and five children in Group 2 attempted and correctly responded to item 21. Consequently, for School A, items 1 through 20 were used as the basis for the present analysis.

The two groups did not differ significantly at pretest, $t(43.37) = -.822$, $p = .415$. Levene's test for equality of variances indicated that the two groups had equivalent variances at midtest and posttest. Consistent with the hypothesis that Group 1, which played Motion Math first, would have higher fractions knowledge relative to Group 2 at the midtest, a one-tailed independent samples t-test was conducted. Results showed that at the midtest, Group 1 had a significantly higher average fractions score compared to Group 2, $t(52) = 2.670$, $p=0.01$. At the posttest, both groups were again equivalent, based on results from a two-tailed independent samples t-test, $t(52) = 0.588$, $p= 0.559$. The effect sizes presented in Table 5 were standardized by taking the average standard deviation for each group for the time points of interest.

Table 5. *Descriptive Data and Effect Sizes for School A Fractions Knowledge*

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Post-Pre	
Group	**n**	**M**	**sd**	**M**	**sd**	**ES**	***t***	**M**	**sd**	**ES**	***t***	**ES**	***T***
1	28	.29	.20	.57	.24	1.27	8.879***	.55	.23	-.08	.752	1.21	7.355***
2	24	.35	.29	.40	.22	.20	2.129	.51	.27	.45	2.954**	0.57	3.962**

To ascertain whether scores at midtest were higher for Group 1 than for Group 2, an ANCOVA was conducted using the pretest as a covariate. Results showed that the effect of treatment condition was significant, $F(1,53) = 25.194$, $p < .001$, η^2 (effect size) = .326. These results show that gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. In other words, controlling for students' initial fractions knowledge, the Motion Math group's midtest score was significantly higher than the control group's. A more stringent comparison of the Motion Math and control group's fractions knowledge was conducted using all three pretest variables as covariates. An ANCOVA test showed that even after controlling for initial levels of fractions knowledge, liking, and self-efficacy, the Motion Math group performed significantly better than did the control group, $F(1,42) = 24.640$, $p < .001$, η^2 (effect size) = .387.

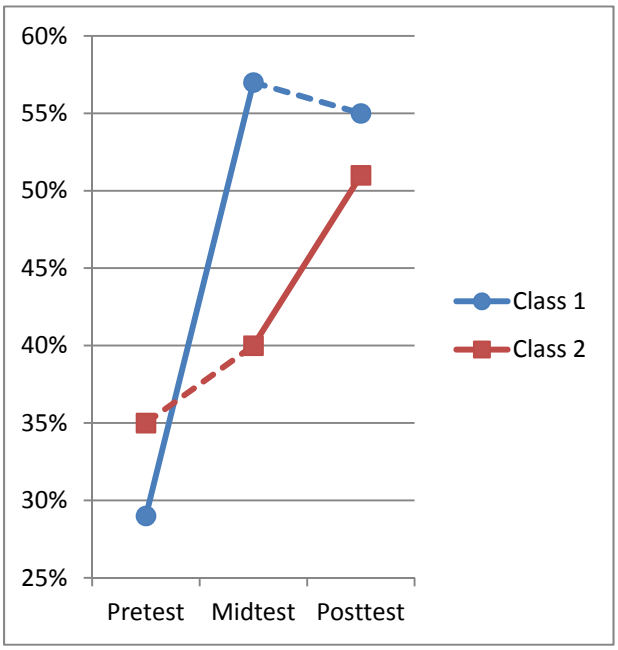


Figure 6. SCHOOL A gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-group gains. Students at School A were given ten minutes to respond to 20 questions.

In addition to comparing the two groups to each other, tests were conducted to determine whether gains within each group were significant, and to determine whether time interacted with treatment condition. Using Bonferroni adjustments to control for Type 1 error, paired sample t-tests for each group showed that the change in scores from pretest to midtest was statistically significant for Group 1, but not for Group 2. From midtest to posttest, the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 5 above. Figure 6 shows the gains by group.

Finally, to examine interaction effects, a repeated measures test was conducted using a General Linear Model. Within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results are consistent with the crossover treatment design of the study and showed that, despite being equivalent at the start and end of the study, the two groups had statistically different test trajectories, $F(1,50) = 29.783$, $p < .001$

.

School B Results

At School B, the fractions test was administered using the iPad device, and students were not timed. Scores for each student were calculated by summing the correct responses and dividing by the total number of questions. The two groups were statistically equivalent at pretest, $t(48) = -.692$, $p = .492$, and the variance at each time point did not differ significantly between groups. One-tailed independent samples t-tests found that the two groups were statistically equivalent at both midtest and posttest. Standardized effect sizes are presented in Table 6.

Table 6. *Descriptive Data and Effect Sizes for School B Fractions Knowledge*

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	**n**	**M**	**sd**	**M**	**sd**	**ES**	***T***	**M**	**sd**	**ES**	***t***	**ES**	***t***
1	26	.31	.144	.38	.209	0.40	2.446*	.40	.214	0.09	0.497	0.50	2.662**
2	24	.34	.168	.33	.181	0.06	0.214	.45	.205	0.62	3.421***	0.59	4.390***

A more robust comparison of the two groups the midtest was conducted using an ANCOVA with pretest scores as covariate. Results showed that the effect of treatment condition was significant, $F(1,53) = 4.427$, $p = .040$, η^2 (effect size) = .078. In other words, gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. A second ANCOVA was conducted that added initial levels of fractions self-efficacy and fractions liking as covariates. The results showed that even after controlling for initial fractions knowledge, self-efficacy, and liking, Group 1 significantly outperformed Group 2 at the midtest, $F(1,53) = 4.069$, $p = .049$, η^2 (effect size) = .075.

Tests were also conducted to ascertain whether gains within each group were significant, and to see whether time interacted with treatment condition. Paired sample t-tests with Bonferroni adjustments showed the same pattern as was obtained for School A. Change in scores from pretest to midtest was statistically significant for Group 1, but not for Group 2. From midtest to posttest,

the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 6 above. Figure 7 shows the gains by group.

Interaction effects were tested with a General Linear Model repeated measures analysis. Within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results support the hypothesis in a crossover design and demonstrated that the two groups had statistically different test trajectories, $F(1,48) = 5.734$, $p = .021$, even though they were equivalent at the start and end of the study.

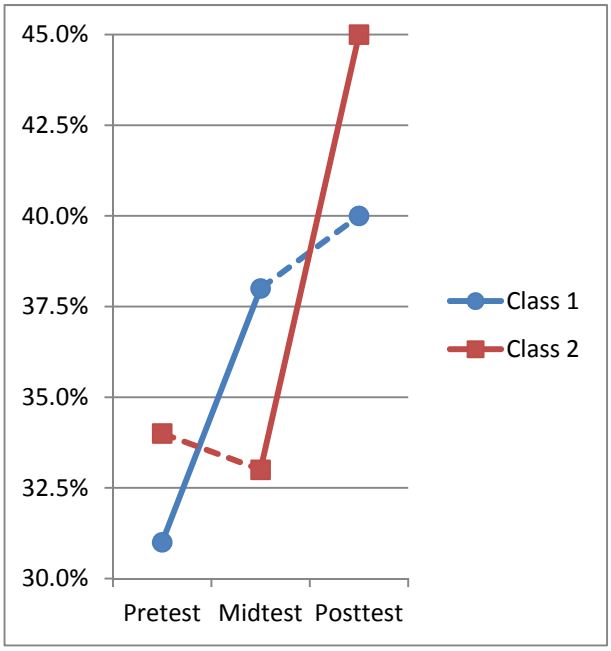


Figure 7. SCHOOL B learning gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-group gains.

Fractions Attitudes: Self-Efficacy, Liking, and Knowledge Self-Ratings

The study also examined changes in students' reported knowledge self-ratings, their self-efficacy for fractions, and their liking of fractions. Similar to the hypotheses for fractions learning, the hypotheses were that students in Group 1 would show increases on average for these variables from pretest to midtest, and remain constant from midtest to posttest. In contrast, Group 2 scores were expected to remain constant from pretest to midtest and to increase significantly from midtest to posttest. Since administrations of the fractions attitudes measures at both school sites were comparable, attitudes data were aggregated.

Fractions Self-Efficacy

The two groups had equivalent self-efficacy scores at the start of the study, $t(92) = -.112$, $p = .911$, and at all three time points did not differ significantly in variance. At the midtest, average self-efficacy for Group 1 was significantly higher than that of Group 2, $t(92) = 2.414$, $p=0.009$. At the posttest, both groups were again equivalent, based on results from a two-tailed independent samples t-test, $t(92) = 0.140$, $p= 0.889$. Standardized effect sizes are shown in Table 7.

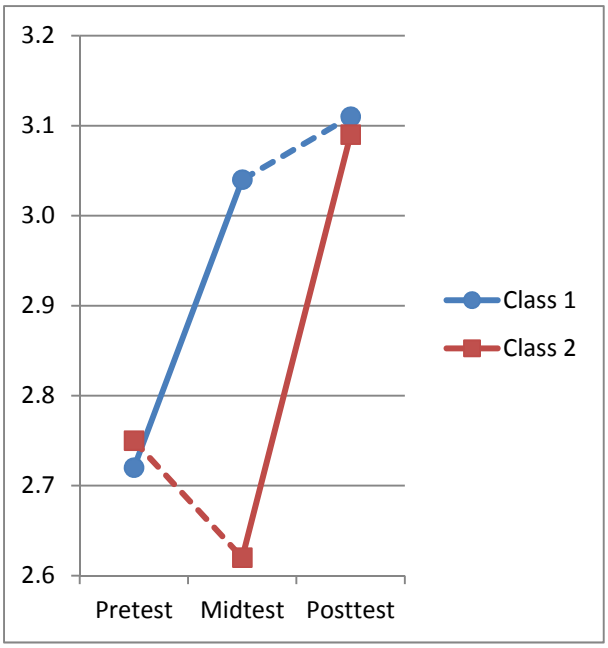


Figure 8. Self-efficacy gains over time for Group 1 (Motion Math first) and Group 2 (Regular first). Solid lines are statistically significant within-group gains.

Controlling for pretest levels of self-efficacy, ANCOVA results showed that the effect of treatment condition was significant, $F(1,94) = 7.357$, $p = .008$. In other words, self-efficacy gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. Analyses were also carried out to determine whether gains within each group were significant. Using Bonferroni adjustments to control for Type 1 error, paired sample t-tests for each group showed that change in scores from pretest to midtest was statistically significant for Group 1, but not for Group 2. From midtest to posttest, the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 7 above. Figure 8 shows the gains by group.

Table 7. *Descriptive Data, Effect Sizes, and Paired-Sample Results for Self-Efficacy*

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	n	M	sd	M	sd	ES	*t*	M	sd	ES	*t*	ES	*t*
1	50	2.72	.838	3.04	.804	0.39	2.057*	3.11	.812	0.09	0.704	0.47	3.176**
2	44	2.75	.737	2.62	.872	0.16	-1.386	3.09	.870	0.54	3.885***	0.42	3.029**

Using a General Linear Model, within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results are consistent with the crossover treatment design of the study and showed that, despite being equivalent at the start and end of the study, the two groups had statistically different trajectories, $F(1,92) = 7.558$, $p = .007$.

Fractions Liking

Analyses for fractions liking were identical to those for self-efficacy. Results are shown in Table 8 and Figure 9. The two groups were statistically equivalent at pretest and posttest, and had

equivalent variance at all three time points. At the midtest, Group 1 posted significantly higher fractions liking than did Group 2, $t(92)= 2.124$, $p = .018$. ANCOVA tests comparing midtest means with initial fractions liking as a covariate yielded a similar pattern of results, $F(1, 97) = 5.324$, $p=.023$. Pairwise comparisons using Bonferroni adjustments within each group showed that Group 1 had statistically significant increases in fractions liking from pretest to midtest, $t(49) = 2.097$, $p = .02$, but not from midtest to posttest. The pattern was reversed for Group 2, with a significant increase from midtest to posttest, $t(43) = 4.530$, $p < .001$, but not from pretest to midtest. In other words, gains in fractions liking were associated with the period in which each group played Motion Math. Further confirming these patterns, quadratic within-subject contrasts were significant for treatment condition, $F(1,92) = 9.571$, $p = .003$, indicating different trajectories for each group.

Table 8. *Descriptive Data, Effect Sizes, and Paired-Sample Results for Fractions Liking*

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	**n**	**M**	**sd**	**M**	**sd**	**ES**	***t***	**M**	**sd**	**ES**	***t***	**ES**	***T***
1	50	3.01	.945	3.28	.916	0.29	2.097*	3.31	.942	0.03	0.280	0.32	2.689**
2	44	2.97	.852	2.86	.985	0.12	-.852	3.44	.830	0.64	4.530***	0.56	3.643***

Fractions Knowledge Self-Ratings

Results for fractions knowledge self-ratings are shown in Table 9 and Figure 10. The two groups were statistically equivalent at pretest and posttest, and had equivalent variance at all three time points. At the midtest, Group 1 reported significantly higher fractions knowledge self-ratings compared to Group 2, $t(102)= 2.124$, $p = .018$. ANCOVA tests comparing midtest means with initial fractions knowledge self-rating as a covariate yielded a marginally significant result, $F(1, 102) = 3.261$, $p=.074$. For both groups, changes in fractions knowledge self-ratings did not differ significantly from pretest to midtest, or from midtest to posttest. However, Group 1 did have overall gains from pretest to posttest. Trajectories were not significantly different for each group on this variable.

Students' Ratings of Motion Math

The third research question asked whether students would rate Motion Math positively after playing it for five consecutive school days. Responses to these questions strongly support students' positive attitudes of the game (Figure 11). One hundred percent of participants reported wanting to

play the game again; nearly 80% strongly endorsed wanting to play the game again. One hundred percent of students also found the game fun, and close to 80% strongly endorsed that statement. In response to the question asking whether their friends would like the game, 95% agreed, and 75% strongly agreed. Finally, 95% of students agreed that the game helped them learn fractions, with nearly two-thirds strongly agreeing.

Table 9. *Descriptive Data, Effect Sizes, and Paired-Sample Results for Fractions Knowledge Self-Rating*

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	n	M	sd	M	sd	ES	*t*	M	sd	ES	*t*	ES	*t*
1	55	3.01	.742	3.35	1.489	0.30	1.623	3.46	1.460	0.07	.446	0.41	2.284*
2	46	3.15	.784	2.92	1.500	0.20	-1.023	3.22	.671	0.28	1.292	0.10	0.495

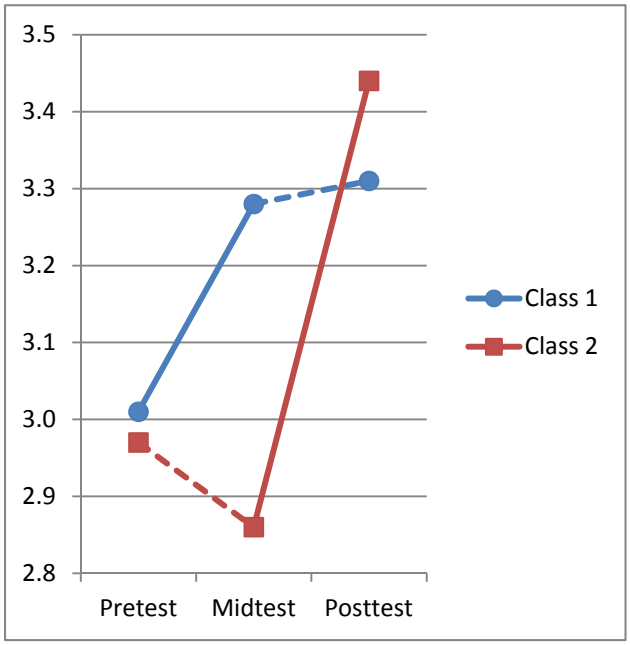


Figure 9. Liking gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-class gains.

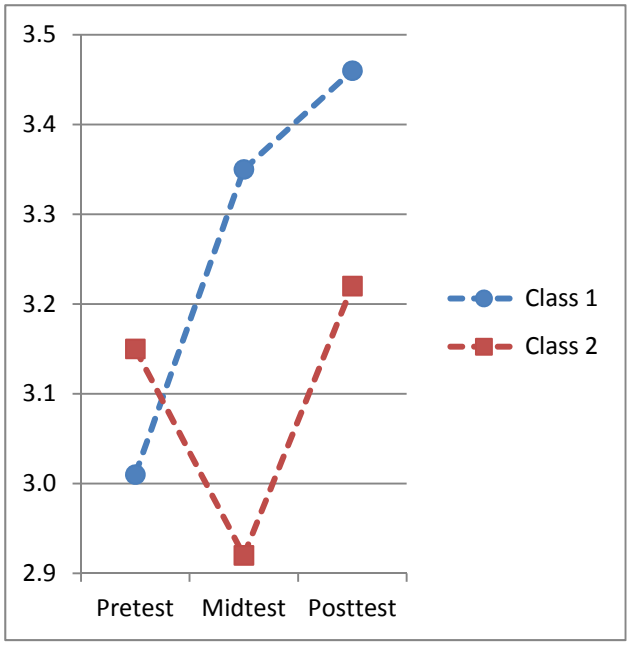
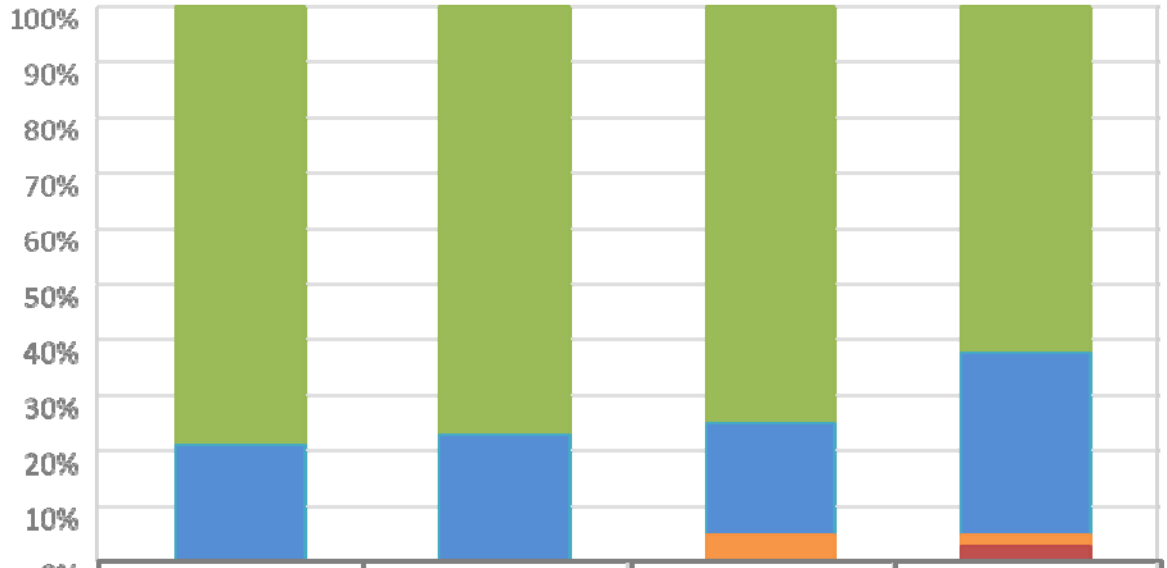


Figure 10. Knowledge self-rating changes over time for Group 1 and Group 2.



	I want to play this game agair	This game was fun	My friends would like this game	This game helped me learr fractions
A lot (4)	79%	78%	75%	62.5%
A little (3)	21%	23%	20%	32.5%
A little (2)	--	--	5%	2.5%
A lot (1)	--	--	--	2.5%
	I don't want to play this game again	This game was boring	My friends would not like this game	This game did not help me learn fractions

Figure 11. Ratings of Motion Math on four indicators

DISCUSSION

Taken together, the results of this study show that Motion Math successfully integrates entertainment value with educational value. The study posed three overarching research questions and hypotheses, which were tested using an experimental repeated-measures crossover design. The data reported here provide solid evidence that a week of daily exposure to Motion Math resulted in improved fractions knowledge and increases in participants' fractions self-efficacy and liking. Moreover, children rated the game very highly. This study is the first to experimentally document learning and motivation gains achieved through iPad game play, and as such offers the field much needed evidence and insight into the potential of mobile apps to support learning.

Fractions Knowledge

At both study sites, significant learning gains were tied to the period in which participants played Motion Math. Lending further strength to the findings is that these patterns were produced by two different measures of fractions knowledge, at schools with two different demographic profiles, and in comparison with a control group. Importantly, the benefits accrued by the first groups to play the game were not lost after several days of non-exposure to the game. Therefore it is reasonable to conclude that the gains made were stable, at least in the short-term.

Why did playing Motion Math improve participants' fractions knowledge? There are many possible answers to this question, and most likely several factors contribute to the game's effectiveness. One possible contributing factor is the instant feedback and scaffolding provided by the game. Another is that players tackle many fractions problems each time they play the game. Over the course of this study, each student encountered an average of over 770 math problems. This represents an incredible amount of fractions practice. The motivation generated by the engaging game mechanic may have been crucial to this benefit. In other words, the entertainment value of the game provided students the motivation necessary to persist in the extensive practice needed to attain mastery and automaticity. The timed nature of the game is another important feature, in that a time constraint adds a sense of urgency to problem-solving, and may also foster rapid schema development. Importantly, the embodied nature of Motion Math game play is a key consideration in its effectiveness. Having to physically tilt the device repeatedly may have helped students develop their mental number line. This explanation is consistent with the Ramani and Siegler (2008), and Siegler and Ramani (2009) studies reviewed earlier.

The success of Motion Math in promoting learning opens up many questions for future exploration regarding the relation between the game design and learning. Important insights would be gained from in-depth analysis of game play data, such as how many problems students correctly solved on the first 'bounce', the trajectories of game level attained, or whether specific problems serve as particularly strong predictors of fractions proficiency. The extent to which assessment of fractions proficiency could be embedded within game play has intriguing implications for student testing.

The gains observed from pretest to posttest indicate that the game served an instructional function. One question that emerges from these results is the extent to which the game was primarily teaching students to relate fractions to the number line, or activating and reinforcing students' prior knowledge. A related question is the extent to which, and the processes by which, experience with fractions estimation transfers to other kinds of fractions knowledge and problem-solving. Future research could tease apart students' conceptual understanding from procedural understanding, and probe the extent to which game play brings students to a deeper conceptual understanding of fractions as numbers. Other topics for additional research include identifying game design features most responsible for learning gains. Do specific problem sequences lead to conceptual insights?

Another set of questions regards ways to optimize learners' interactions with Motion Math. For example, is the game most effective for students who are at a certain level of understanding of fractions? One of the participating teachers in the present study observed that students who had not yet grasped the basic idea of a fraction did not appear to improve their game play, whereas other students gave signs of making new connections and insights. An interesting consideration is how slight modifications to the game might lead to improved learning for a broader range of students. Motion Math was designed for students with a baseline understanding of the fractions representation form. For students with limited prior knowledge who may not naturally extract patterns from game play, introducing explicit sequences may prove effective. For instance, an introductory level might start with a sequence of unit fractions with increasing denominators (i.e., 1/2, 1/3, 1/4, 1/5, 1/6) and explicitly draw the player's attention to the inverse relationship between the denominator and the distance from zero on the number line. Other instructional sequences include using a constant denominator with an incremental decrease or increase in the numerator; or maintaining the same fraction while changing the labels on the number line endpoints. Adding a whole number "refresher" level could serve as an effective scaffold to bridge students' intuitive

understanding of whole numbers to an understanding of fractions as numbers (Wu, 2008). Changing the number line more often may foster more flexible schema development. Motion Math already includes levels in which the number line spans from -1 to zero. Including additional levels with number lines that span zero, such as one anchored at -1 and +1, could help reinforce the concept that fraction magnitude is independent of sign.

Wu (2008) asserted that fractions research should be integrated with the study of fractions instruction. Though Motion Math was created as a stand-alone game, it may be that even minimal instructional support exponentially increases the educational value of game play. Simple instructional prompts from a teacher, parent, or older peer could unlock insights for students. Future research should explore the value added by a variety of instructional wrap-arounds. Another consideration is the length of exposure to the game. This study took place over ten school days. Over longer periods of time, do students develop deeper conceptual understanding?

Fractions Self-Efficacy, Liking, and Knowledge Self-Ratings

The second set of study hypotheses pertained to students' attitudes toward fractions. The results for fractions self-efficacy and liking were highly similar to those for learning. Increases in these attitudes were associated with the period in which students played Motion Math, and persisted after game play. These results are arguably just as important as the learning results, given the strong connection documented in the research literature between students' attitudes and subsequent learning. Developing positive affect for fractions should benefit students as they encounter new challenges in mathematics.

Gains in students' self-efficacy show that their confidence in their ability to solve fractions problems improved as a result of playing Motion Math. Importantly, the study asked students to rate their self-efficacy for fractions in general, not for their ability to succeed at the game. In other words, these outcomes suggest that the experience of playing Motion Math gave students a sense of confidence in fractions that extends beyond game play. One explanation for this outcome is the fact that mastery experiences are a well documented source of self-efficacy (Usher & Pajares, 2009). The design of Motion Math enabled students to experience and be rewarded for success each time they correctly solved a problem. This stands in contrast to the kind of feedback students receive in school, where they are scored on overall assignments or tests. In this context, correctly solving a single problem is unlikely to contribute to a student's self-efficacy. A game environment also promotes persistence, a key motivation indicator. Several features of game play are likely to

contribute to persistence. First, failure is an accepted part of game play, whereas in school tasks it is often reason for students to withdraw from the task. Everyone expects to have to fail many times before winning in a game. Second, a well balanced game maintains the right proportion of challenge to skill level, so that persistence is rewarded by new successes, which in turn fuels more persistence. The game also gives students multiple chances to succeed at each problem. Related to this is that scaffolding communicates to the student that the game is "on their side" and thus encourages risk taking and persistence. Finally, the entertainment value of the game is probably responsible to a large degree for students' persistence.

Students also reported liking fractions more after playing Motion Math. Similar to the results obtained for learning and self-efficacy, these increases remained when students no longer played the game. In a game environment, the positive experience of game play is likely to become associated with the material, much like, in a negative fashion, bad experiences with math class lead to negative affect toward math. One reason for increased liking is the fact that learners tend to have more positive attitudes toward topics they feel they understand and can succeed in. Many schoolchildren believe that fractions are too hard to understand. By offering students opportunities to succeed at solving fractions problems, Motion Math also provided students with experiences that contributed to more positive fractions affect. The importance of this outcome cannot be underestimated for students' future decisions to persist in and succeed at mathematics.

In contrast, students' self-ratings of their fractions knowledge did not yield significant changes, although there were significant differences between the two groups at the midpoint of the study. There are several possible explanations for this outcome. Reliability was very low (α=0.58) for this three-item measure, which was created for the present study to assess whether students might rate themselves more highly after game play on their ability to solve the specific kinds of questions played in the game. The low reliability may be the reason for the nonsignificant changes observed here. It may also be that the instrument, beyond being unreliable, was not a valid measure of self-rated knowledge due to language comprehension. Many students from School A provided inconsistent responses to the negatively worded item for this measure. Another possible explanation is that prior to playing Motion Math, students had never been asked to place a fraction or decimal on the number line, and therefore did not understand the questions. The fact that reliabilities increased to acceptable levels after the experience of game play suggest that once students began playing the game, the question made more sense and they were able to respond more accurately and consistently.

Ratings of Motion Math

Students' ratings of Motion Math were overwhelmingly positive and demonstrate that the game is successful in providing students with an entertaining experience. Outcomes regarding enjoyment of the game and interest in playing it again are all the more powerful in light of the evidence of learning documented here. After playing the game for 20 minutes a day, five days in a row, students' enthusiasm for the game remained high. The fact that students still wanted to play the game after doing so for five consecutive school days suggests that the game mechanic has hit the mark with a powerful 'addictive' quality. Perhaps most importantly, in Motion Math, learning is seamlessly integrated with assessment. Game play and assessment are one and the same. Future research could probe the keys to the game's entertainment value, including interactive features, visual, haptic, and audio rewards, the tilt-based game mechanic, and the hand-held nature of device interaction.

LIMITATIONS

Like every research study, the present investigation had several limitations that should be considered in interpreting and applying the results. First, the study did not compare Motion Math to traditional classroom instruction or to other educational apps. Therefore, the study did not enable conclusions to be drawn regarding the effectiveness of Motion Math relative to other approaches to learning fractions. Second, the study did not collect information on individual student demographics or academic achievement. Therefore it was not possible to examine whether certain groups of students (e.g., by gender, ethnicity, prior knowledge) responded more favorably than others to the Motion Math game. Third, game play data were not analyzed in this study. As a result, although the data enable valid claims about *whether* students learned, we do not know *why* they learned. Moreover, it is highly likely that certain students benefited more than others from exposure to the game. Finally, it could be argued that Motion Math was successful in promoting students' fractions learning because of the large number of problems students solved. In other words, if Motion Math is essentially fractions practice, then research should be conducted to determine whether it is more effective than other kinds of practice, for instance worksheets or flashcards.

Another consideration is the effect the iPad itself may have had on the outcomes obtained here. Anecdotal evidence suggested that most students in this study did not own their own iPad. It is therefore possible that students' reported enthusiasm for fractions and for Motion Math could be explained in part by their enthusiasm for the device itself. Even if the iPad was responsible for the

attitudes outcomes, it is unlikely that the use of the iPad *per se* was responsible for fractions learning. The excitement at using the iPad may have given students the motivation they needed to persist at the game, which in turn led to fractions learning. The question is whether students would continue to engage effectively with the game once the novelty of the technology subsides. Finally, the fact that students participated in the context of the school day may have influenced the quality of their engagement with the game. It is possible that the quality of engagement in informal settings would result in smaller learning and attitudes gains. The present study offers a foundation for future research into these issues.

RECOMMENDATIONS

Based on the results reported here, together with informal observations of students' game play during the study, several recommendations are offered for further study and game enhancements of Motion Math, as well as for the field of mobile learning apps.

Motion Math Applications and Game Design

1. For children whose understanding of fractions representations is just developing, adding an additional introductory game level may boost the learning potential of the game. Students with limited fractions knowledge, and who generally struggle with mathematics, may benefit from explicit patterns, such as systematically increasing or decreasing the denominator or numerator. An optional refresher level with whole numbers may also prove effective for helping students recognize the relation between fractions magnitude and whole number magnitude.
2. The current game includes "pincher" levels with reduced scaffolding, so that players do not learn to rely entirely on the hints to succeed. The game may be more effective if pincher levels are introduced more frequently.
3. Future versions of Motion Math could include opportunities for students and their teachers or parents to track their progress on specific types of fractions. The game could also be modified to respond more specifically to each player's strengths and weaknesses, for instance by challenging students more often with types of fractions they have not yet mastered. This could be taken a step further by responding instructionally via sequences that support students in developing schema for the kinds of fractions they have yet to understand.
4. Currently, when players lose at a level, they must restart the game from the first level. This could undermine students' motivation, especially as they approach the final game levels and the earlier

ones are not sufficiently challenging. Updates to the game could impose a penalty of a certain number of levels in relation to the level attained.

5. Given well documented stereotypes linking males more strongly to math and science relative to females, the game designers should consider adding feedback from a female voice as well.

Further Research on Motion Math

1. This study examined the effectiveness of Motion Math as a standalone learning activity. Future research should examine whether complementing Motion Math with instructional supports can lead to even greater gains in students' fractions learning and attitudes.
2. Given the positive learning outcomes in the present study, additional research could examine game play data for trends linked to improved fractions knowledge. Studies spanning a longer time period would also be better poised to ascertain whether over time students develop deeper conceptual understanding of fractions as numbers which can be placed on a number line. Observations also suggest that the game may be most beneficial when students are at specific developmental stages in their understanding of fractions. Future research could compare initial levels of fractions knowledge with game play data to identify interaction patterns and optimal game learning conditions.
3. The teachers from both schools in this study indicated that many students asked to use the "challenge a friend" feature of the game. To maintain consistency of the intervention, they were asked to refrain from doing so. However, additional research could examine the effect that social gameplay has on students' fractions learning and attitudes.
4. The internet connectivity of the iPad device offers powerful opportunities to conduct research that compares the effectiveness of variations of the game. By serving different versions of the game to players, research could be conducted to determine which feedback algorithms or instructional sequences are more effective, and for which learners.

Research and Development of Mobile Learning Apps

Beyond recommendations specific to Motion Math development and research, the findings from this study have implications for the research and development of mobile learning apps more generally.

1. Emergent mobile devices make it possible for players to interact physically with devices in revolutionary ways. Much is yet to be learned about how the new affordances of these devices

can support learning, and how our conception of learning itself might change in light of these new ways of interacting with the world around us.

2. With attention to protection to individuals' privacy, research could use GPS data to determine where people are most likely to make use of mobile learning apps. Are they typically at home, work, waiting in line at the grocery store? Does location of use relate to aspects of mobile learning app usage? How is location of game play related to game features and to learning? For example, it may be that individuals are less likely to engage in games that require conspicuous physical interaction in public spaces. In addition, brief games are perhaps most appropriate for learning that happens best with distributed practice, whereas games that focus on shifting strong misconceptions may demand longer play periods, and consequently be less aligned with spontaneous usage.
3. Much is to be learned about how individual differences affect the way that learners interact physically with mobile devices. In the present study, differences were observed in the extent to which individuals were willing to dramatically tilt the device. While some students tended to use exaggerated movements, others were reticent to engage physically with the device. Future research, and the development of embodied learning apps, should consider the interplay between individual characteristics and game features.
4. Mobile learning apps offer tremendous potential for dynamic assessment that is embedded in game play. In other words, in well designed apps, learning could be tracked without the need for external tests and quizzes. Research and development is already underway in this direction, however this approach to assessment is yet in its infancy.
5. Motion Math is grounded in research and theory on the field of mathematics and on how people learn. To be effective, developers of learning apps will need to invest substantial resources to deeply probe the target learning material, concepts, and skills, and explicitly shape game mechanics to reflect the target learning.

CONCLUSION

The well-documented difficulties children face when learning fractions call for new approaches to teaching this topic. To date, the present study is the first to hold the promise of new technologies up to experimental evidence. This project investigated Motion Math, a mobile learning game that leverages the physical interaction features of the iPad to engage students in a fractions learning experience. In this controlled experiment, Motion Math significantly improved participants' fractions knowledge and attitudes. Moreover, children's ratings of the game were quite positive. This study thus offers evidence that Motion Math successfully integrates learning and entertainment, and in the process boosts players' attitudes towards fractions.

REFERENCES

Alibali, M. W., & Nathan, M. J. (2011). Embodiment in mathematics teaching: Evidence from learners' and teachers' gestures. *Journal of the Learning Sciences*, DOI:10.1080/10508406.2011.611446

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*, 181-185.

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, *84*, 191-215.

Behr, M., Lesh, R., Post, T., & Silver, E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematical concepts and processes* (pp. 91–125). New York: Academic Press.

Clark, A. (1999). An embodied cognitive science? *Trends In Cognitive Sciences*, *3*(9), 345-351.

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*(1), 155-159.

Department of Education. (1997). *Mathematics equals opportunity*. White Paper prepared for U.S. Secretary of Education Richard W. Riley.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*(1–2), 43–74.

Habgood, M. P. J., & Ainsworth, S. E. (2011): Motivating children to learn effectively: Exploring the value of intrinsic integration in educational games. *Journal of the Learning Sciences*, *20*(2), 169-20.

Hart, K. (2000). Mathematics content and learning issues in the middle grades. In *Mathematics Education in the Middle Grades : Proceedings of a National Convocation and Action Conferences* (pp. 50-57). Washington, DC: National Academy Press.

Harter, S. (1981). A new self-report scale of intrinsic versus extrinsic orientation in the classroom: Motivational and informational components. *Developmental Psychology*, *17*, 300-312.

Hidi, S., & Renninger, K. A. (2006). The four-phase model of interest development. *Educational Psychologist*, *41*(2), 111–127.

Hiebert, J. (1985). Children's knowledge of common and decimal fractions. *Education and Urban Society*, *17*(4), 427–437.

Hiebert, J., & Tonnessen, L. H. (1978). Development of the fraction concept in two physical contexts: An exploratory investigation. *Journal for Research in Mathematics Education*, *9*(5), 374–378.

Johnson-Glenberg, M. C., Birchfield, D., Savvides, P. & Megowan-Romanowicz, C. (2010) Semi-virtual embodied learning – real world STEM assessment. In L. Annetta & S. Bronack (Eds.) *Serious Educational Game Assessment: Practical Methods and Models for Educational Games, Simulations and Virtual Worlds* (pp. 225-241). Sense Publications, Rotterdam.

Malone, T. W., & Lepper, M. R. (1987). Making learning fun: A taxonomy of intrinsic motivations for learning. In R.E. Snow & M.J. Farr (Eds.), *Aptitude, Learning and Instruction III: Conative and Affective Process Analyses* (pp. 223-253). Hillsdale, N.J.: Erlbaum.

Misquitta, R. (2011). A review of the literature: Fraction instruction for struggling learners in mathematics. *Learning Disabilities Research & Practice*, *26*(2), 109–119.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Newton, K. J. (2008). An extensive analysis of preservice teachers' knowledge of fractions. *American Educational Research Journal*, *45*(4), 1080-1110.

Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research*, *66*(4), 543-578.

Princiotta, D., Flanagan, K. D., and Germino Hausken, E. (2006). *Fifth Grade: Findings From The Fifth-Grade Follow-up of the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K).* (NCES 2006-038) U.S. Department of Education. Washington, DC: National Center for Education Statistics.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable inprovements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*, 375-394.

Riconscente, M. M. (2010, September). Using latent profile analysis to evaluate the 4-Phase Model of Interest Development. In M. Ainley (Chair) *The next decade of interest research: Processes and measures*. Symposium presented at the biannual International Conference on Motivation, Porto, Portugal.

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide* (NCEE #2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from whatworks.ed.gov/ publications/practiceguides.

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games -- but not circular ones -- improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, *101*, 545-560.

Siegler, R. S., Thompson, C. A., & Schneider M. (2011B, April). Magnitude representations are central to understanding of fractions. In Kroesbergen, E. ,& Kolkman, M. (Chairs), *Domain General and Domain Specific Processes in Numerical Development*. Symposium presented at the biannual meeting of the Society for Research in Child Development. Montreal, Quebec.

Siegler, R. S., Thompson, C. A., Schneider, M. (2011A). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273-296.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education*, *29*(4), 465-488.

Tolentino, L., Birchfield, D., Megowan-Romanowicz, C., Johnson-Glenberg, M. C., Kelliher, A., & Martinez, C. (2009). Teaching and learning in the mixed-reality science classroom. *Journal of Science Education and Technology*, *18*, 501-517.

Usher, E. L., & Pajares, F. (2008). Sources of self-efficacy in school: Critical review of the literature and future directions. *Review of Educational Research*, *78*, 751-796.

Wigfield, A., & Eccles, J. S. (2000). Expectancy‐value theory of achievement motivation. *Contemporary Educational Psychology*, *25*, 68‐81.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, *9*(4), 625-636.

Wong, M., & Evans, D. (2007, July). Students' conceptual understanding of equivalent fractions. In J. Watson & K. Beswick, *Mathematics: Essential Research, Essential Practice, Vol 1*. Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia (pp. 824–833). Hobart, Australia.

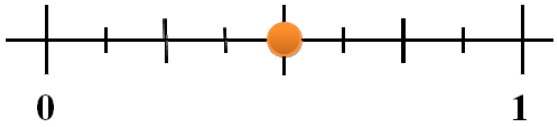
Wu, H. (2008). *Fractions, decimals, and rational numbers*. Retrieved from http://math.berkeley.edu/~wu/NMPfractions4.pdf

Wu, H. (2010, April). Teaching fractions: Is it poetry or mathematics? Paper presented at the 2010 National Council of Teachers of Mathematics Annual Meeting and Exposition, San Diego, CA.

APPENDIX A: SCHOOL A FRACTIONS KNOWLEDGE TEST (PAPER; TIMED)

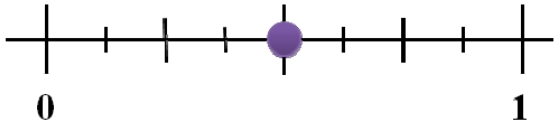
On the number line below, a ● shows where $\frac{1}{2}$ is.

Draw another ● to show where $\frac{3}{4}$ is.



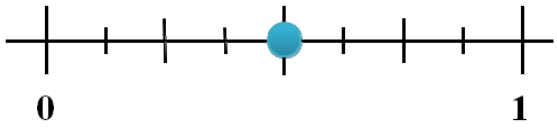
On the number line below, a ● shows where $\frac{1}{2}$ is.

Draw another ● to show where $\frac{10}{10}$ is.

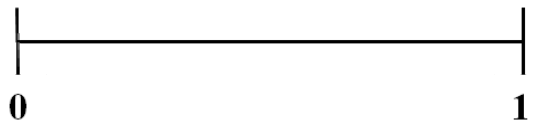


On the number line below, a ● shows where $\frac{1}{2}$ is.

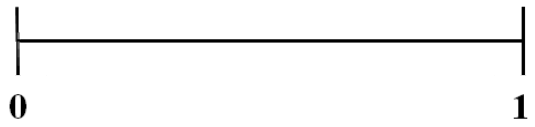
Draw another ● to show where $\frac{1}{8}$ is.



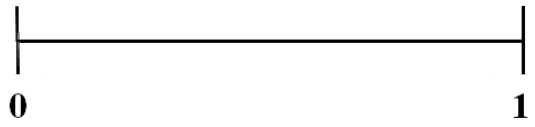
Draw a ● to show where $\frac{1}{8}$ is on the number line.



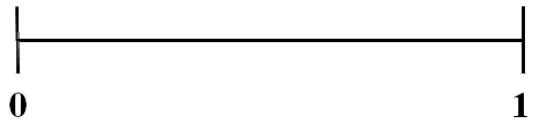
Draw a ● to show where $\frac{2}{4}$ is on the number line.



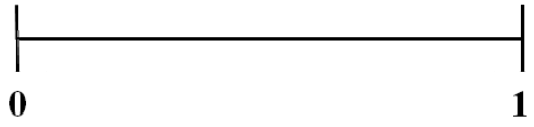
Draw a ● to show where $\frac{6}{12}$ is on the number line.



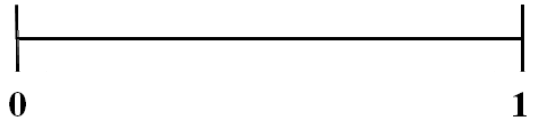
Draw a ● to show where $\frac{2}{5}$ is on the number line.



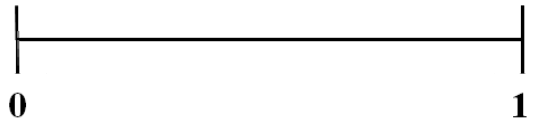
Draw a ● to show where 0.80 is on the number line.



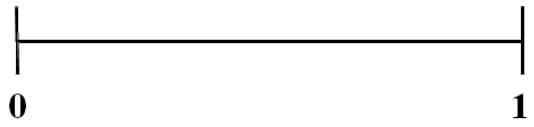
Draw a ● to show where 0.50 is on the number line.



Draw a ● to show where 0.10 is on the number line.



Draw a ● to show where 0.60 is on the number line.

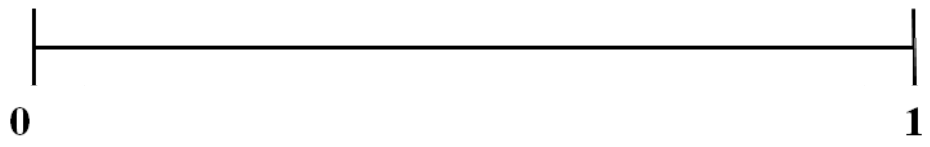


On the number line below,

draw a ○ to show where $\frac{1}{3}$ is,

draw a □ to show where $\frac{1}{4}$ is,

and draw a △ to show where $\frac{1}{2}$ is.

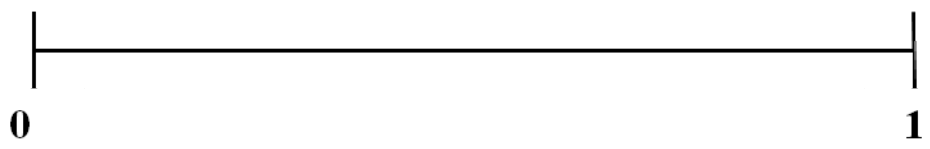


On the number line below,

draw a ○ to show where $\frac{1}{5}$ is,

draw a □ to show where $\frac{5}{5}$ is,

and draw a △ to show where $\frac{3}{5}$ is.

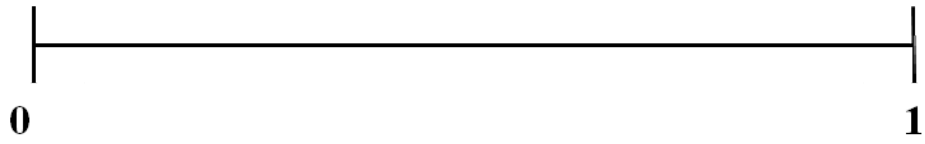


On the number line below,

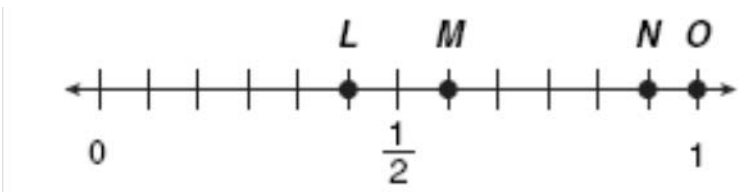
draw a ○ to show where $\frac{5}{6}$ is,

draw a □ to show where $\frac{2}{3}$ is,

and draw a △ to show where $\frac{1}{2}$ is.



Which point is located at $\frac{7}{12}$ on the number line below?



Circle your answer: ***L*** ***M*** ***N*** ***O***

1. Which fraction is BIGGER? $\frac{5}{8}$ *or* $\frac{3}{8}$

2. Which fraction is BIGGER? $\frac{3}{2}$ *or* $\frac{4}{3}$

3. Which fraction is BIGGER? $\frac{11}{12}$ *or* $\frac{10}{11}$

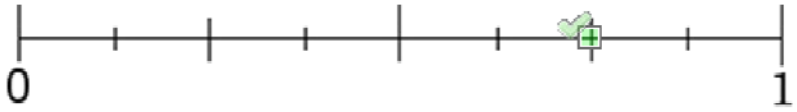
4. Which fraction is BIGGER? $\frac{4}{7}$ *or* $\frac{4}{8}$

5. Which fraction is BIGGER? $\frac{2}{6}$ *or* $\frac{1}{2}$

APPENDIX B: SCHOOL B FRACTIONS KNOWLEDGE TEST (iPAD; UNTIMED)

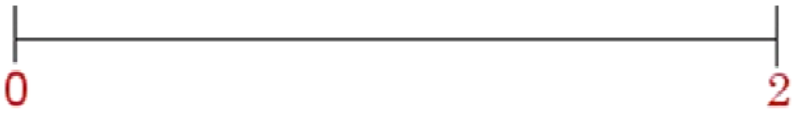
Representative Sample of Items

CLICK on the number line to show where $\frac{3}{4}$ goes.



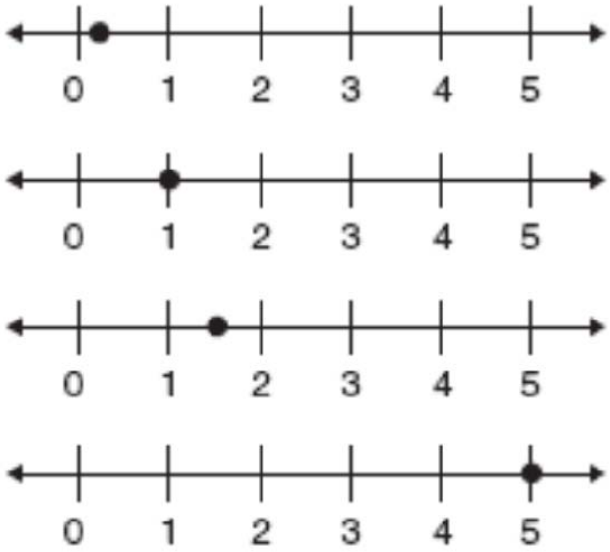
CLICK on the number line to show where $\frac{10}{10}$ is.

CLICK on the number line to show where $\frac{4}{4}$ is.

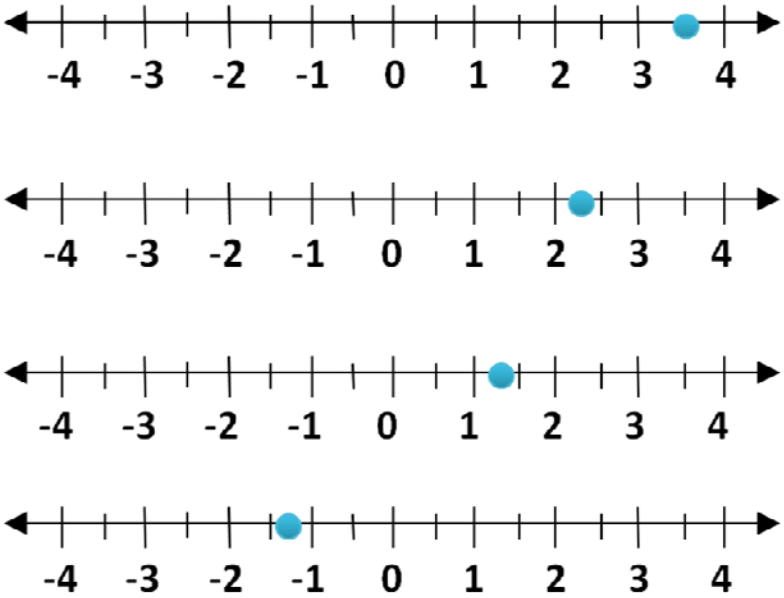


CLICK on the number line to show where $\frac{5}{3}$ is.

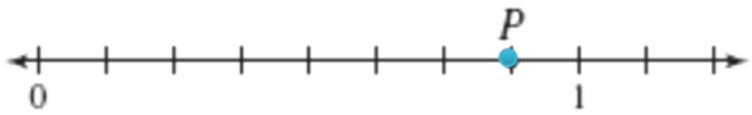
Which number line appears to show the dot placed at $\frac{1}{5}$?



Which point on the number line best represents 1.35?



What fraction is best represented by point P on this number line?



$\frac{1}{8}$ $\frac{1}{5}$ $\frac{3}{4}$ $\frac{7}{8}$

What fraction has a value closest to $\frac{1}{2}$?

$\frac{5}{8}$ $\frac{1}{6}$ $\frac{2}{2}$ $\frac{1}{5}$

Which number represents the shaded part of the figure?

- 2.8
- 0.5
- 0.2
- 0.02